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An Approach to Higher-Order Assertion-based Debugging of Higher-Order (C)LP Programs *

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Abstract. Higher-order constructs extend the expressiveness of first-order (Constraint) Logic Programming ((C)LP) both syntactically and semantically. At the same time assertions have been in use for some time in (C)LP systems helping programmers detect errors and validate programs. However, these assertion-based extensions to (C)LP have not been integrated well with higher-order to date. This paper contributes to filling this gap by extending the assertion-based approach to error detection and program validation to the higher-order context within (C)LP. We propose an extension of properties and assertions as used in (C)LP in order to be able to fully describe arguments that are predicates. The extension makes the full power of the assertion language available when describing higher-order arguments. We provide syntax and semantics for (higher-order) properties and assertions, as well as for programs which contain such assertions, including the notions of error and partial correctness. We also discuss several alternatives for performing run-time checking of such programs.

1 Introduction

Higher-order programming adds flexibility to the software development process. Within the (Constraint) Logic Programming ((C)LP) paradigm, Prolog has included higher-order constructs since the early days, and there have many other proposals for combining the first-order kernel of (C)LP with different higher-order constructs (see, e.g., [1,2,3,4,5,6]). Many of these proposals are currently in use in different (C)LP systems and have been found very useful in programming practice, inheriting the well-known benefits of code reuse (templates), elegance, clarity, and modularization.

A number of extensions have also been proposed for (C)LP in order to enhance the process of error detection and program validation. In addition to the

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use of classical strong typing [7,8], a number of other approaches have been proposed which are based on the dynamic and/or static checking of user-provided, optional *assertions* [9,10,11,12,13,14,15,16]. In practice, different aspects of the model of [13,15] have been incorporated in a number of widely-used (C)LP systems, such as Ciao, SWI, and XSB [17,18,19]. A similar evolution is represented by the soft/gradual typing-based approaches in functional programming and the contracts-based extensions in object-oriented programming [20,21,22,23,24].

These two aspects, assertions and higher-order, are not independent. When higher-order constructs are introduced in the language it becomes necessary to describe properties of arguments of predicates that are themselves also predicates. While the combination of contracts and higher-order has received some attention in functional programming [25,26], within (C)LP the combination of higher-order with the previously mentioned assertion-based approaches has received comparatively little attention to date. Current Prolog systems simply use basic atomic types (i.e., stating simply that the argument is a `pred`, `callable`, etc.) to describe predicate-bearing variables. The approach of [27] is oriented to meta programming. It allows describing meta-types but there is no notion of directionality (modes), and only a single pattern is allowed per predicate.

This paper contributes to filling the existing gap between higher-order and assertions in (C)LP. Our starting point is the Ciao assertion model [13,15], since, as mentioned before, it has been adopted at least in part in a number of the most popular (C)LP systems. After some preliminaries and notation (Section 2) we start by extending the traditional notion of programs and derivations in order to deal with higher-order calls and recall and adapt the notions of first-order conditional literals, assertions, program correctness, and run-time checking to this type of derivations (Section 3). This part allows us to revisit the traditional model in this new, higher-order context, while introducing a different formalization than the original one of [13]. This formalization, which will be used throughout the paper, is more compact and gathers all assertion violations as opposed to just the first one, among other differences. We then define an extension of the properties used in assertions and of the assertions themselves to higher-order, and provide corresponding semantics and results (Section 4).

2 Preliminaries and Notation

We recall some concepts and notation from standard (C)LP theory. We denote by VS, FS, and PS the set of variable, function, and predicate symbols, respectively. Variables start with a capital letter. Each $p \in \mathsf{PS}$ and $f \in \mathsf{FS}$ is associated to a natural number called its *arity*, written $\mathsf{ar}(p)$ or $\mathsf{ar}(f)$. The set of terms TS is inductivelly defined as follows: $\mathsf{VS} \subset \mathsf{TS}$, if $f \in \mathsf{FS}$ and $t_1, \ldots, t_n \in \mathsf{TS}$ then $f(t_1, \ldots, t_n) \in \mathsf{TS}$ where $\mathsf{ar}(f) = n$. An *atom* has the form $p(t_1, ..., t_n)$ where $p \in PS$, $\mathsf{ar}(p) = n$, and $t_1, ..., t_n \in TS$. A *constraint* is essentially a conjunction of expressions built from predefined predicates (such as term equations or inequalities over the reals) whose arguments are constructed using predefined functions (such as real addition). A *literal* is either an atom or a constraint. A

goal is a finite sequence of literals. A *rule* is of the form H :- B where H, the *head*, is an atom and B, the *body*, is a possibly empty finite sequence of literals. A *constraint logic program*, or *program*, is a finite set of rules.

We use σ to represent a variable renaming and $\sigma(X)$ to represent the result of applying the renaming σ to some syntactic object X (a term, atom, literal, goal, etc.). The *definition* of an atom A in a program, $\mathsf{defn}(A)$, is the set of variable renamings of the program rules such that each renaming has A as a head and has distinct new local variables. We assume that all rule heads are normalized, i.e., H is of the form $p(X_1, ..., X_n)$ where the $X_1, ..., X_n$ are distinct free variables. This is not restrictive since programs can always be normalized, and it facilitates the presentation. However, for conciseness in the examples we sometimes use non-normalized programs. Let $\bar{\exists}_L \theta$ be the constraint θ restricted to the variables of the syntactic object L. We denote *constraint entailment* by $\models$, so that $\theta_1 \models \theta_2$ denotes that θ_1 entails θ_2. In such case we say that θ_2 is *weaker* than θ_1.

For brevity, we will assume in the rest of the paper that we are dealing with a single program, so that all sets of rules, etc. refer to that implicit program and it is not necessary to refer to it explicitly in the notation.

2.1 Operational Semantics

The operational semantics of a program is given in terms of its "derivations," which are sequences of reductions between "states." A *state* $\langle G \mid \theta \rangle$ consists of a goal G and a constraint store (or *store* for short) θ. We use :: to denote concatenation of sequences and we assume for simplicity that the underlying constraint solver is complete. We use $S \rightsquigarrow S'$ to indicate that a reduction can be applied to state S to obtain state S'. Also, $S \rightsquigarrow^* S'$ indicates that there is a sequence of reduction steps from state S to state S'. We denote by $D_{[i]}$ the i-th state of the derivation. As a shorthand, given a non-empty derivation D, $D_{[-1]}$ denotes the last state. A *query* is a pair (L, θ), where L is a literal and θ a store, for which the (C)LP system starts a computation from state $\langle L \mid \theta \rangle$. The set of all derivations from the query Q is denoted $\mathsf{derivs}(Q)$. The observational behavior of a program is given by its "answers" to queries. A finite derivation from a query (L, θ) is *finished* if the last state in the derivation cannot be reduced. Note that $\mathsf{derivs}(Q)$ contains not only finished derivations but also all intermediate derivations from a query. A finished derivation from a query (L, θ) is *successful* if the last state is of the form $\langle \square \mid \theta' \rangle$, where $\square$ denotes the empty goal sequence. In that case, the constraint $\bar{\exists}_L \theta'$ is an *answer* to S. We denote by $\mathsf{answers}(Q)$ the set of answers to a query Q. A finished derivation is *failed* if the last state is not of the form $\langle \square \mid \theta \rangle$. A query Q *finitely fails* if $\mathsf{derivs}(Q)$ is finite and contains no successful derivation.

3 First-order Assertions on Higher-order Derivations

3.1 Higher-order Programs and Derivations

We start by extending the definition of program, state reduction, and derivations in order to deal with the syntax and semantics of higher-order calls.

Definition 1 (Higher-order Programs). Higher-order programs *are a generalization of* constraint logic programs *where:*

- *The set of literals* LS *is extended to include* higher-order literals $X(t_1, \ldots, t_n)$, *where* $X \in \mathsf{VS}$ *and the* $t_i \in \mathsf{TS}$.
- *The set of terms* TS *is extended so that* $\mathsf{PS} \subset \mathsf{TS}$ *(i.e., predicate symbols* p *can be used as constants).*

In the following we assume a simple semantics where when a call to a higher-order literal $X(t_1, \ldots, t_n)$ occurs, X has to be constrained to a predicate symbol in the store:[3]

Definition 2 (Reductions in Higher-order Programs). *A state* $S = \langle L :: G \mid \theta \rangle$ *where* L *is a literal can be* reduced *to a state* S', *denoted* $S \rightsquigarrow S'$, *as follows:*

1. *If* L *is a constraint and* $\theta \wedge L$ *is satisfiable, then* $S' = \langle G \mid \theta \wedge L \rangle$.
2. *If* L *is an atom of the form* $p(t_1, \ldots, t_n)$, *for some rule* $(L \texttt{:-} B) \in \mathsf{defn}(L)$, *then* $S' = \langle B :: G \mid \theta \rangle$.
3. *If* L *is of the form* $X(t_1, \ldots, t_n)$, *then* $S' = \langle G' \mid \theta \rangle$ *where:*

$$G' = \begin{cases} p(t_1, \ldots, t_n) :: G & \text{if } \exists p \in \mathsf{PS} \wedge \theta \models (X = p) \wedge \mathsf{ar}(p) = n \\ \epsilon_{uninst\_call} & \text{otherwise} \end{cases}$$

The concepts of answers and of finished and successful derivations carry over without change to this notion of higher-order derivations. The notion of (finitely) failed derivation is extended as follows:

Definition 3 ((Finitely) Failed Derivation). *A finished derivation from a query* (L, θ) *is* failed *iff its last state is not of the form* $\langle \Box \mid \theta' \rangle$ *or* $\langle \epsilon_{uninst\_call} \mid \theta \rangle$.

Finally, we introduce the concept of *floundered* derivations:

Definition 4 (Floundered Derivation). *A finished derivation from a query* (L, θ) *is* floundered *iff its last state is of the form* $\langle \epsilon_{uninst\_call} \mid \theta \rangle$.

[3] This is also the most frequent semantics in current systems. Other alternatives, such as residuation [28] (delays), predicate enumeration, etc. can also be used, requiring relatively straightforward adaptations of the model proposed.

3.2 First-order *Pred* Assertions

Assertions are linguistic constructions for expressing properties of programs. They are used for detecting deviations of the program behavior (symptoms) with respect to such assertions, or to ensure that no such deviations exist (correctness). Herein, we will use the *pred* assertions of [29], given that they are the most frequently used assertions in practice, and they subsume the other assertion schemas in that language. Thus, in the following we will use simply the term assertion to refer to a pred assertion. Assertions allow specifying certain conditions on the constraint store that must hold at certain points of program derivations. In particular, they allow stating sets of *preconditions* and *conditional postconditions* for a given predicate. A set of assertions for a predicate is of the form:

$$\texttt{:- pred } Head \texttt{ : } Pre_1 \texttt{ => } Post_1\texttt{.}$$
$$\ldots$$
$$\texttt{:- pred } Head \texttt{ : } Pre_n \texttt{ => } Post_n\texttt{.}$$

where $Head$ is a normalized atom that denotes the predicate that the assertions apply to, and the Pre_i and $Post_i$ refer to the variables of $Head$. We assume that variables in assertions are renamed such that the $Head$ atom is identical for all assertions for a given predicate. A set of assertions as above states that in any execution state $\langle Head :: G \mid \theta \rangle$ at least one of the Pre_i conditions should hold, and that, given the $(Pre_i, Post_i)$ pair(s) where Pre_i holds, then, if $Head$ succeeds, the corresponding $Post_i$ should hold upon success. The following example illustrates the basic concepts involved:

Example 1. The procedure `qsort(A,B)` is the usual one that relates lists `A` and their sorted versions `B`. The following assertions:

```
:- pred qsort(A,B) : list(A) => (sorted(B), list(B)).
:- pred qsort(A,B) : list(B) => (permutation(B,A), list(A)).
```

state that (restrict the meaning of `qsort` to):

- `qsort(A,B)` should be called either with `A` constrained to a list or with `B` constrained to a list;
- if `qsort(A,B)` succeeds when called with `A` constrained to a list then on success `B` should be a sorted list;
- if `qsort(A,B)` succeeds when called with `B` constrained to a list then on success `A` should be a list which is a permutation of `B`.

3.3 Conditions on the Constraint Store

The conditions on the constraint store used in assertions are specified by means of special literals (e.g., `list(A)`, `sorted(B)`, `list(B)`, and `permutation(B,A)` in the previous example) that we will herein call *prop* literals. More concretely, we assume the Pre_i and $Post_i$ to be DNF formulas of such literals.

We also assume that for each prop literal L_p used in some assertion there exists a corresponding predicate p defining it. Then, we can define the meaning of prop literals as follows:

Definition 5 (Meaning of a *Prop* Literal). *The meaning of a prop literal L_p defined by predicate p, denoted $|L_p|$, is the set of constraints given by* answers$((L_p, true))$.

Intuitively, the meaning of prop literals is the set of "weakest" constraints for which the literal holds:

Example 2. Prop literals `list/1` and `sorted/1` can be defined by:

```
list([]).                 sorted([]).          sorted([_]).
list([_|L]) :- list(L).   sorted([X,Y|L]) :-  X =< Y, sorted([Y|L]).
```

Then, their meaning is given by:

$$|list(A)| = \{A = [], A = [B|C] \wedge list(C)\}, \text{ and}$$
$$|sorted(A)| = \{A = [], A = [B], A = [B, C|D] \wedge B \leq C \wedge E = [C|D] \wedge sorted(E)\}.$$

The following definition from [13] defines when the condition represented by a prop literal (defined by a program predicate) holds for a given store:

Definition 6 (Succeeds Trivially). *A prop literal L* succeeds trivially *for θ, denoted $\theta \Rightarrow_P L$, iff $\exists \theta' \in$ answers$((L, \theta))$ such that $\theta \models \theta'$. A DNF formula of prop literals succeeds trivially for θ if all of the prop literals of at least one conjunct of the formula succeeds trivially.*

Intuitively, a prop literal L succeeds trivially if L succeeds for θ without adding new "relevant" constraints to θ:

Example 3. Consider prop literals $list(A)$ and $sorted(B)$ and the predicate definitions of Example 2:

- Assume that $\theta = (A = f)$. Since $\forall \theta' \in |list(A)| : \theta \not\models \theta'$, as we would expect, $\theta \not\Rightarrow_P list(A)$.
- Assume now that $\theta = (A = [\_|Xs])$. Though A is compatible with a list, it is not actually a (nil terminated) list. Again in this case $\forall \theta' \in |list(A)| : \theta \not\models \theta'$ and thus again $\theta \not\Rightarrow_P list(A)$. The intuition behind this is that we cannot guarantee that A is actually a list given θ, since a possible instance of A in θ is $A = [\_|f]$, which is clearly not a list.
- Finally, assume that $\theta = (A = [B] \wedge B = 1)$. In such case $\exists \theta' = (A = [B|C] \wedge C = [])$ such that $\theta \models \theta'$ and $\exists c = (B = 1)$ such that $(c \wedge \theta' \not\models false) \wedge (\theta' \wedge c \models \theta)$. Thus, in this last case $\theta \Rightarrow_P list(A)$.

This means that we are considering prop literals as *instantiation* checks [30,29]: they are true iff the variables they check for are at least as constrained as their predicate definition requires.

Definition 7 (Test Literal). *A prop literal L is a* test *iff $\forall \theta$ either $\theta \Rightarrow_P L$ or (L, θ) finitely fails.*

3.4 First-order Assertion Conditions and their Semantics

We represent the different checks on the constraint store imposed by a set of assertions as a set of *assertion conditions* as follows.

Definition 8 (Assertion Conditions for a Predicate). *Given a predicate represented by a normalized atom $Head$, if the corresponding set of assertions is $\mathcal{A} = \{A_1 \ldots A_n\}$, with $A_i =$ "`:- pred` $Head$ `:` Pre_i `=>` $Post_i$`."` the set of* assertion conditions *for $Head$ is $\{C_0, C_1, \ldots, C_n\}$, with:*

$$C_i = \begin{cases} \mathit{calls}(Head, \bigvee_{j=1}^{n} Pre_j) & i = 0 \\ \mathit{success}(Head, Pre_i, Post_i) & i = 1..n \end{cases}$$

If there are no assertions associated with $Head$ then the corresponding set of conditions is empty. The set of assertion conditions for a program is the union of the assertion conditions for each of the predicates in the program. Also, given a single assertion A_i we define its corresponding set of assertion conditions as $\{C_0, C_i\}$ (this will be useful in defining the status of an assertion).

The $\mathsf{calls}(Head, \ldots)$ conditions encode the checks that the calls to the predicate represented by $Head$ are within those admissible by the set of assertions, and we thus call them the *calls assertion conditions*. The $\mathsf{success}(Head_i, Pre_i, Post_i)$ conditions encode the checks for compliance of the successes for particular sets of calls, and we thus call them the *success assertion conditions*.

Example 4. The assertion conditions corresponding to the predicate assertions for `qsort` in Example 1 are as follows:

$$\mathsf{calls}(qsort(A, B), (list(A), list(B)))$$
$$\mathsf{success}(qsort(A, B), list(A), (sorted(B), list(B)))$$
$$\mathsf{success}(qsort(A, B), list(B), (permutation(B, A), list(A)))$$

In order to define the semantics of assertion conditions, we introduce the auxiliary partial functions prestep and step as follows:

$$\mathsf{prestep}(L_a, D) = (\theta, \sigma) \equiv D_{[-1]} = \langle L :: G \mid \theta \rangle \wedge \exists \sigma \; L = \sigma(L_a)$$
$$\mathsf{step}(L_a, D) = (\theta, \sigma, \theta') \equiv D_{[-1]} = \langle G \mid \theta' \rangle \; \wedge \exists i \; D_{[i]} = \langle L :: G \mid \theta \rangle \wedge \exists \sigma \; L = \sigma(L_a)$$

Given a derivation whose current state is a call to L_a (normalized atom), the prestep function returns the substitution σ for L_a, and the constraint store θ at the predicate *call* (i.e., just before the literal is reduced). Given a derivation whose current state corresponds exactly to the return from a call to L_a, the step function returns the substitution σ for L_a, the constraint store θ at the call to L_a, and the constraint store θ' at L_a's *success* (i.e., just after all literals introduced from the body of L_a have been fully reduced). Using these functions, the semantics of our *calls* and *success* assertion conditions are given by the following definition:

Definition 9 (Valuation of an Assertion Condition on a Derivation). *Given a* calls *or* success *assertion condition C, the* valuation of C on a derivation D, *denoted* $\mathsf{solve}(C, D)$ *is defined as follows:*

$$\begin{aligned} \mathsf{solve}(\mathsf{calls}(L_a, Pre), D) &\equiv (\mathsf{prestep}(L_a, D) = (\theta, \sigma)) \Rightarrow (\theta \Rightarrow_P \sigma(Pre)) \\ \mathsf{solve}(\mathsf{success}(L_a, Pre, Post), D) &\equiv (\mathsf{step}(L_a, D) = (\theta, \sigma, \theta')) \Rightarrow \\ &\qquad ((\theta \Rightarrow_P \sigma(Pre)) \Rightarrow (\theta' \Rightarrow_P \sigma(Post))) \end{aligned}$$

where L_a is a normalized atom.

3.5 Status of Assertions and Partial Correctness

As mentioned before, the intended use of our assertions is to perform debugging with respect to partial correctness, i.e., to ensure that the program does not produce unexpected results for *valid* ("expected") queries.[4] Thus, we extend our notion of program to include assertions and valid queries.

Definition 10 (Annotated Program). *An annotated program is a tuple $(P, \mathcal{Q}, \mathcal{A})$ where P is a (higher-order)* constraint logic program *(as defined in Section 2), $\mathcal{Q}$ is a set of valid queries, and $\mathcal{A}$ is a set of assertions. As before, $\mathcal{A}_C$ denotes the set of* calls *and* success *assertion conditions derived from $\mathcal{A}$.*

In the context of annotated programs we extend *derivations* to operate on the set of valid queries as follows: $\mathsf{derivs}(\mathcal{Q}) = \bigcup_{Q \in \mathcal{Q}} \mathsf{derivs}(Q)$. We now provide several simple definitions which will be instrumental:

Definition 11 (Assertion Condition Status). *Given the set of queries $\mathcal{Q}$, the assertion condition C can be either* checked *or* false*, as follows:*

$$\begin{aligned} \mathsf{checked}(C) &\equiv \forall D \in \mathsf{derivs}(\mathcal{Q}) \ . \ \mathsf{solve}(C, D) \\ \mathsf{false}(C) &\equiv \exists D \in \mathsf{derivs}(\mathcal{Q}) \mid \neg \mathsf{solve}(C, D) \end{aligned}$$

Definition 12 (Assertion Status). *In an annotated program $(P, \mathcal{Q}, \mathcal{A})$ an assertion $A \in \mathcal{A}$ is* checked *(*false*) if all (any) of the corresponding assertion conditions are* checked *(*false*).*

Definition 13 (Partial Correctness). *An annotated program $(P, \mathcal{Q}, \mathcal{A})$ is* partially correct *w.r.t. the set of assertions $\mathcal{A}$ and the set of queries $\mathcal{Q}$ iff $\forall A \in \mathcal{A}$, A is checked for $\mathcal{Q}$.*

Note that it follows immediately that a program is partially correct if all its assertion conditions are checked. The goal of assertion checking is thus to determine whether each assertion A is false or checked for $\mathcal{Q}$. Again, for this it is sufficient to prove the corresponding assertions conditions false or checked. There are two kinds of approaches to doing this (which can also be combined).

[4] In practice, this set of expected queries is determined from module interfaces that define the set of exported predicates.

While it is in general not possible to try all derivations stemming from $\mathcal{Q}$, an alternative is to explore a hopefully representative set of them [16]. Though this does not allow fully validating the program in general, it makes it possible to detect many incorrectness problems. This approach is explored in Section 3.6 in the context of our higher-order derivations. The second approach is to use global analysis techniques and is based on computing safe approximations of the program behavior statically [11,15]. The extension of this approach to higher-order assertions is beyond the scope of this paper.

3.6 Operational Semantics for Higher-order Programs with First-order Assertions

We now provide an operational semantics which checks whether assertion conditions hold or not while computing the (possibly higher-order) derivations from a query.

Definition 14 (Labeled Assertion Condition Instance). *Given the atom L_a and the set of assertion conditions $\mathcal{A}_C$, $\mathcal{A}_C^{\#}(L_a)$ denotes the set of* labeled *assertion condition instances for L_a of the form $c\#C_a$, such that $\exists C \in \mathcal{A}_C$, $C = \mathsf{calls}(L, Pre)$ (or $C = \mathsf{success}(L, Pre, Post)$), σ is a renaming s.t. $L = \sigma(L_a)$, $C_a = \mathsf{calls}(L_a, \sigma(Pre))$ (or $C_a = \mathsf{success}(L_a, \sigma(Pre), \sigma(Post))$), and c is an identifier that is unique for each C_a.*

In order to keep track of the violated assertion conditions, we introduce an extended program state of the form $\langle G \mid \theta \mid \mathcal{E} \rangle$, where $\mathcal{E}$ denotes the set of identifiers for falsified assertion condition instances. We also extend the set of literals with syntactic objects of the form $\mathsf{check}(c)$ where c is an identifier for an assertion condition instance, which we call *check literals*. Thus, a *literal* is now a constraint, an atom, a higher-order literal, or a check literal.[5]

Definition 15 (Reductions in Higher-order Programs with First-order Assertions). *A state $S = \langle L :: G \mid \theta \mid \mathcal{E} \rangle$, where L is a literal can be* reduced *to a state S', denoted $S \rightsquigarrow_{\mathcal{A}} S'$, as follows:*

1. *If L is a constraint or $L = X(t_1, \ldots, t_n)$, then $S' = \langle G' \mid \theta' \mid \mathcal{E} \rangle$ where G' and θ' are obtained in a same manner as in $\langle L :: G \mid \theta \rangle \rightsquigarrow \langle G' \mid \theta' \rangle$*
2. *If L is an atom and $\exists (L \texttt{:-} B) \in \mathsf{defn}(L)$, then $S' = \langle B :: PostC :: G \mid \theta \mid \mathcal{E}' \rangle$ where:*

$$\mathcal{E}' = \begin{cases} \mathcal{E} \cup \{\bar{c}\} & \text{if } \exists\, c\#\mathsf{calls}(L, Pre) \in \mathcal{A}_C^{\#}(L) \text{ s.t. } \theta \not\Rightarrow_P Pre \\ \mathcal{E} & \text{otherwise} \end{cases}$$

and $PostC$ is the sequence $\mathsf{check}(c_1) :: \ldots :: \mathsf{check}(c_n)$ including all the checks $\mathsf{check}(c_i)$ such that $c_i\#\mathsf{success}(L, Pre_i, Post_i) \in \mathcal{A}_C^{\#}(L) \ \wedge \ \theta \Rightarrow_P Pre_i$.

[5] While check literals are simply instrumental here, note that they are also directly useful for supporting program point assertions (which are basically check literals that appear in the body of rules) [29]. However, for simplicity we do not discuss program point assertions in this paper.

3. If L is a check literal $\mathsf{check}(c)$, then $S' = \langle G \mid \theta \mid \mathcal{E}' \rangle$ where:

$$\mathcal{E}' = \begin{cases} \mathcal{E} \cup \{\bar{c}\} & \text{if } c\#\mathsf{success}(L, \_, Post) \in \mathcal{A}^{\#}_{C}(L) \wedge \theta \not\Rightarrow_P Post \\ \mathcal{E} & \text{otherwise} \end{cases}$$

Note that the order in which the PostC check *literals are selected is irrelevant.*

The set of derivations for a program from its set of queries $\mathcal{Q}$ using the semantics with assertions is denoted $\mathsf{derivs}_{\mathcal{A}}(\mathcal{Q})$.

Definition 16 (Error-erased Derivation). *The set of* error-erased *derivations from $\rightsquigarrow_{\mathcal{A}}$ is obtained by a syntactic rewriting $(-)^\circ$ that removes states that begin by a check literal, check literals from goals, and the error set. It is recursively defined as follows:*

$$\{D_1, \ldots, D_n\}^\circ = \{D_1^\circ, \ldots, D_n^\circ\}$$

$$(S_1, \ldots, S_m, S_{m+1})^\circ = \begin{cases} (S_1, \ldots, S_m)^\circ & \text{if } S_{m+1} = \langle \mathsf{check}(\_) :: \_ \mid \_ \mid \_ \rangle \\ (S_1, \ldots, S_m)^\circ \parallel ((S_{m+1})^\circ) & \text{otherwise} \end{cases}$$

$$\langle G \mid \theta \mid \mathcal{E} \rangle^\circ = \langle G^\circ \mid \theta \rangle$$

$$(L :: G)^\circ = \begin{cases} G^\circ & \text{if } L = \mathsf{check}(\_) \\ L :: (G^\circ) & \text{otherwise} \end{cases}$$

$$\square^\circ = \square$$

where $\parallel$ stands for sequence concatenation.

Theorem 1 (Correctness and Completeness Under Assertion Checking). *For any annotated program $(P, \mathcal{Q}, \mathcal{A})$, given $\mathcal{D} = \mathsf{derivs}(\mathcal{Q})$ and $\mathcal{D}' = \mathsf{derivs}_{\mathcal{A}}(\mathcal{Q})$, it holds that $\mathcal{D}$ and $\mathcal{D}'$ are equivalent after filtering out check literals and error sets (formally defined as $\mathcal{D} = (\mathcal{D}')^\circ$ in Def. 16).*

Proof. We will prove $\mathcal{D} = (\mathcal{D}')^\circ$ by showing that $\mathcal{D} \subseteq (\mathcal{D}')^\circ$ and $\mathcal{D} \supseteq (\mathcal{D}')^\circ$.

- ($\subseteq$) For all $D \in \mathcal{D}$ exists $D' \in \mathcal{D}'$ so that $D = (D')^\circ$.
- ($\supseteq$) For all $D' \in \mathcal{D}'$, $D = (D')^\circ \in \mathcal{D}$.

We will prove each case:

- ($\subseteq$) Let $D = (S_1, \ldots, S_n)$, $S_i = \langle L_i \mid \theta_i \rangle$, for some $Q = (L_1, \theta_1) \in \mathcal{Q}$ and $S_i \rightsquigarrow S_{i+1}$. Proof by induction on the length n of D:
  - Base case ($n = 1$). Let $S'_1 = \langle L_1 \mid \theta_1 \mid \emptyset \rangle$. It holds that $(S'_1)^\circ = \langle L_1 \mid \theta_1 \mid \emptyset \rangle)^\circ = \langle L_1^\circ \mid \theta_1 \rangle = \langle L_1 \mid \theta_1 \rangle = S_1$ (since L_1 does not contain any check literal). Thus, $(D')^\circ = ((S'_1))^\circ = ((S'_1)^\circ) = (S_1) = D$.
  - Inductive case (show $n{+}1$ assuming n holds). For each $D_2 = (S_1, \ldots, S_n, S_{n+1})$ there exists $D'_2 = (S'_1, \ldots, S'_m, S'_{m+1})$ such that $(D'_2)^\circ = D_2$. Given the induction hypothesis it is enough to show that for each $S_n \rightsquigarrow S_{n+1}$ there exists $S'_m \rightsquigarrow_{\mathcal{A}} S'_{m+1}$, such that $(S'_{m+1})^\circ = S_{n+1}$. According to $\rightsquigarrow_{\mathcal{A}}$ (see Def. 15), L'_{m+1} and θ'_{m+1} are obtained in the same way than in $\rightsquigarrow$ (see Def. 2), except for the introduction of check literals. Since all check literals are removed in error-erased states, it follows that $(S'_{m+1})^\circ = S_{n+1}$. $\square$

– ($\supseteq$) Let $D' = (S'_1, \ldots, S'_m)$, $S'_i = \langle L'_i \mid \theta'_i \mid \mathcal{E}_i \rangle$, for some $Q = (L'_1, \theta'_1) \in \mathcal{Q}$ and $S'_i \rightsquigarrow_{\mathcal{A}} S'_{i+1}$. Proof by induction on the length m of D':
  - Base case ($m = 1$). It holds that $(S'_1)^\circ = S_1$ (showed in base case for $\subseteq$). Then $(D')^\circ = D \in \mathcal{D}$.
  - Inductive case (show $m+1$ assuming m holds). We want to show that given $D'_2 = (S'_1, \ldots, S'_m, S'_{m+1})$, $(D'_2)^\circ = D_2 \in \mathcal{D}$. Given the induction hypothesis it is enough to show that for each $S'_m \rightsquigarrow_{\mathcal{A}} S'_{m+1}$ there exists $S_n \rightsquigarrow S_{n+1}$ such that $S_{n+1} = (S'_{m+1})^\circ$ (so that $(S_1, \ldots, S_n, S_{n+1}) \in \mathcal{D}$) or $S_n = (S'_{m+1})^\circ$ ($D_2 = D \in \mathcal{D}$). According to cases of Def. 15:
    * If L'_m begins with a check literal then $(L'_{m+1})^\circ = (L'_m)^\circ$. Thus $(S'_{m+1})^\circ = (S'_m)^\circ = S_n$.
    * Otherwise, it holds that $(S'_{m+1})^\circ = S_{n+1}$ using the same reasoning than in the inductive case for $\subseteq$. □

This result implies that the semantics with assertions can also be used to obtain all answers to the original query. Furthermore, the following theorem guarantees that we can use the proposed operational semantics for annotated programs in order to detect (all) violations of assertions:

Definition 17 (Run-time Valuations of an Assertion Condition on a Derivation). *Let $\mathcal{E}(D)$ denote the error set of the last state of derivation D, $D_{[-1]} = \langle \_ \mid \_ \mid \mathcal{E} \rangle$. The run-time valuation of an assertion condition C on a derivation D is given by:*

$$\mathit{rtsolve}(C, D) \equiv \forall c, C', \sigma, L \ (c\#C' \in \mathcal{A}^{\#}_C(L) \wedge \sigma(C) = C') \Rightarrow \mathcal{E}(D) \nvdash \bar{c}$$

I.e., condition $\mathsf{rtsolve}(C, D)$ is valid if none of the possible instances of the assertion condition C are in the error set for derivation D.

Theorem 2 (Run-time Error Detection). *For any annotated program $(P, \mathcal{Q}, \mathcal{A})$, $C \in \mathcal{A}_C$ is false iff $\exists\, D \in \mathit{derivs}_{\mathcal{A}}(\mathcal{Q})$ s.t. $\neg \mathit{rtsolve}(C, D)$.*

Proof. $A \in \mathcal{A}_C$ is false
$\Leftrightarrow$ from Def. 12 and Def. 8 $\exists \{C_c, C_s\}$ assertion conditions s.t. $\mathsf{false}(C_c) \vee \mathsf{false}(C_s)$, where $C_c = \mathsf{calls}(L, Pre)$ and $C_s = \mathsf{success}(L, Pre, Post)$ correspond to A. Let us first prove $\neg\mathsf{rtsolve}(C_c, D)$, and then $\neg\mathsf{rtsolve}(C_s, D)$.

$\mathsf{false}(C_c)$
$\Leftrightarrow$ from Def. 11 $\exists D \in \mathsf{derivs}(\mathcal{Q})$ s.t. $\neg\mathsf{solve}(C_c, D)$
$\Leftrightarrow$ from Def. 9 ($\mathsf{prestep}(L, D) = (\theta, \sigma) \wedge \theta \not\Rightarrow_P \sigma(Pre)$)
$\Leftrightarrow$ from Def. 15 $\exists\ S \rightsquigarrow_{\mathcal{A}} S'$ where:

$$S = \langle L :: G \mid \theta \mid \mathcal{E} \rangle \text{ s.t. } \exists\ c\#\mathsf{calls}(L, Pre) \in \mathcal{A}^{\#}_C(L)$$
$$S' = \langle \_ \mid \theta \mid \mathcal{E}' \rangle \wedge \mathcal{E}' = \mathcal{E} \cup \{\bar{c}\}$$

$\Leftrightarrow$ from Def. 17 $\neg\mathsf{rtsolve}(C_c, D)$ □

$\mathsf{false}(C_s)$

$\Leftrightarrow$ from Def. 11 $\exists D \in \mathsf{derivs}(\mathcal{Q})$ s.t. $\neg\mathsf{solve}(C_s, D)$
$\Leftrightarrow$ from Def. 9 $(\mathsf{step}(L, D) = (\theta, \sigma, \theta') \wedge \theta \Rightarrow_P \sigma(Pre) \wedge \theta' \not\Rightarrow_P \sigma(Post))$
$\Leftrightarrow$ from Def. 15 $\exists\ S \rightsquigarrow^*_{\mathcal{A}} S' \rightsquigarrow_{\mathcal{A}} S''$ where:

$$S = \langle L :: G \mid \theta \mid \_ \rangle \wedge \exists\ c\#\mathsf{success}(L, Pre, Post) \in \mathcal{A}^{\#}_C(L) \wedge \theta \Rightarrow_P Pre$$
$$S' = \langle \mathsf{check}(c) :: G \mid \theta' \mid \mathcal{E}' \rangle \wedge \theta' \not\Rightarrow_P Post$$
$$S'' = \langle \_ \mid \_ \mid \mathcal{E}'' \rangle \wedge \mathcal{E}'' = \mathcal{E}' \cup \{\bar{c}\}$$

$\Leftrightarrow$ from Def. 17 $\neg\mathsf{rtsolve}(C_s, D)$ □

Th. 2 states that assertion condition C is false iff there is a derivation D in which the run-time valuation of the assertion condition of C in D is false (i.e., if at least one instance of the assertion condition A is in the error set for such derivation D). Given a set of $false$ assertion conditions we can easily derive the set of $false$ assertions using Def. 8. In order to prove that any assertion is checked this has to be done for all possible derivations for all possible queries, which is often not possible in practice. This is why analysis based on abstractions is often used in practice for this purpose.

4 Higher-order Assertions on Higher-order Derivations

Once we have established basic results for the case of first-order assertions in the context of higher-order derivations, we extend the notion of assertion itself to the higher-order case. The motivation is that in the higher-order context terms can be bound to predicates and our aim is to also be able to state and check properties of such predicates.

4.1 Anonymous Assertions

We start by generalizing the notion of assertion to include *anonymous assertions*: assertions where the predicate symbol is a variable from VS, which can be instantiated to any suitable predicate symbol from PS to produce non-anonymous assertions. An anonymous assertion is an expression of the from "`:- pred` L `:` Pre `=>` $Post$", where L is of the form $X(V_1, \ldots, V_n)$ and Pre and $Post$ are DNF formulas of *prop* literals.

Example 5. The anonymous assertion "`:- pred X(A,B) : list(A) => list(B).`" states that any predicate $p \in P$ that X is constrained to should be of arity 2, it should be called with its first argument instantiated to a list, and if it succeeds, then its second argument should be also a list on success.

We now introduce *predprops*, which gather a number of anonymous assertions in order to fully describe variables containing higher-order terms (predicate symbols), similarly to how *prop* literals describe conditions for variables containing first-order terms.

Definition 18 (Predprop). *Given* Pre_i *and* $Post_i$ *conjunctions of* prop *literals, a* predprop $pp(X)$ *is an expression of the form:*

$$\begin{array}{l}\texttt{pp(X)\{ :- pred } X(V_1,\ldots,V_m) \texttt{ : } Pre_1 \texttt{ => } Post_1.\\ \quad\ldots\\ \quad\texttt{:- pred } X(V_1,\ldots,V_m) \texttt{ : } Pre_n \texttt{ => } Post_n.\ \texttt{\}}\end{array}$$

Definition 19 (Anonymous Assertion Conditions for a *predprop*). *The corresponding set of* anonymous assertion conditions *for the predprop* $pp(X)$ *is defined as* $\mathcal{A}_C[pp(X)] = \{C_i[X] \mid i = 0..n\}$ *where:*

$$C_i[X] = \begin{cases} \mathit{calls}(X(V_1,\ldots,V_m), Pre) & i = 0 \\ \mathit{success}(X(V_1,\ldots,V_m), Pre_i, Post_i) & i = 1..n \end{cases}$$

The variable X *can be instantiated to a particular predicate symbol* $q \in \mathsf{PS}$ *to produce a set of non-anonymous assertion conditions* $\mathcal{A}_C[pp(p)]$ *for* q*.*

Example 6. Consider defining a `comparator(Cmp)` *predprop* that describes predicates of arity 3 which can be used to compare numerical values:

```
comparator(Cmp) {
  :- pred Cmp(X,Y,Res) : int(X),int(Y) => between(-1,1,Res).
  :- pred Cmp(X,Y,Res) : flt(X),flt(Y) => between(-1,1,Res). }.
```

The `comparator(Cmp)` *predprop* includes two anonymous assertions describing a set of possible preconditions and postconditions for predicates of this kind. In this example:

$$\begin{array}{l}\mathcal{A}_C[comparator(Cmp)] = \{\\ \quad \mathsf{calls}(Cmp(X,Y,Res), (int(X) \wedge int(Y)) \vee (flt(X) \wedge flt(Y))),\\ \quad \mathsf{success}(Cmp(X,Y,Res), int(X) \wedge int(Y), between(-1,1,Res))\\ \quad \mathsf{success}(Cmp(X,Y,Res), flt(X) \wedge flt(Y), between(-1,1,Res))\ \}\end{array}$$

Example 7. Fig. 1 provides a larger example. This example is more stylized for brevity, but it covers a good subset of the relevant cases, used later to illustrate the semantics.

Definition 20 (Meaning of a *predprop* Literal). *The* meaning *of a* predprop $pp(X)$*, denoted* $|pp(X)|$ *is the set of constraints* $\{X = q \mid q \in \mathsf{PS}, \forall\_\#C \in \mathcal{A}_C[pp(q)] : \mathit{checked}(C)\}$.

A predicate given by its predicate symbol $p \in \mathsf{PS}$ is *compatible* with a *predprop* $pp(X)$ if all the assertions resulting from $pp(p)$ are *checked* for all possible queries in an annotated program.

4.2 Operational Semantics for Higher-order Programs with Higher-order Assertions

We now discuss several alternative operational semantics for higher-order programs with higher-order assertions. In all cases the aim of the semantics is to check whether assertions with *predprops* hold or not during the computation of the derivations from a query.

```
:- nneg(P) { :- pred P(X) => nnegint(X). }.
:- neg(P)  { :- pred P(X) =>  negint(X). }.

:- pred test_c(P,N) : nneg(P).
:- pred test_c(P,N) :  neg(P).
test_c(P,N) :- P(N).

:- pred test_s(N,P) : nnegint(N) => nneg(P).
:- pred test_s(N,P) :  negint(N) =>  neg(P).
test_s( 1,P) :- P = z. % bug here, should be P = p
test_s(-1,P) :- P = n.

z(1). z(-2).   p(1). p(2).   n(-1). n(-2).   c(a). c(b).
```

Fig. 1. Sample Program with predprops.

Checking with Static *predprops* According to Definition 20, a *predprop* literal $pp(X)$ denotes the subset of predicates for which all the associated assertions are checked. When that set of assertions can be statically computed, then $\theta \Rightarrow_P Cond$ can be used for both prop and predprop $Cond$ literals, and the operational semantics is identical to the one for the higher-order programs and regular assertions.

We will denote as $S \rightsquigarrow_{H\mathcal{A}_s} S'$ a reduction from a state S to a state S' under the semantics for higher-order derivations in programs with assertions that may contain higher-order properties, which are statically precomputed. Thus, state reductions are performed as follows:

$$\frac{\langle G \mid \theta \mid \mathcal{E} \rangle \rightsquigarrow_{\mathcal{A}} \langle G' \mid \theta' \mid \mathcal{E}' \rangle}{\langle G \mid \theta \mid \mathcal{E} \rangle \rightsquigarrow_{H\mathcal{A}_s} \langle G' \mid \theta' \mid \mathcal{E}' \rangle}$$

The meaning of each predprop, $|pp(X)|$, can be inferred or checked (if given by the user) by static analysis.

In this semantics, given the program shown in Fig. 1 and the goal `test_c(z,-2)`, assertions are detected to be false since $\{P = \mathtt{z}\} \not\subset |\mathtt{neg(P)}|$ and $\{P = \mathtt{z}\} \not\subset |\mathtt{nneg(P)}|$.

Checking with Dynamic *predprops* Given the difficulty in determining the meaning of $|pp(X)|$ statically, we also propose a semantics with dynamic checking. We start with an over-approximation of each predprop $|pp(X)| = \{X = p \mid p \in \mathsf{PS}\}$ and incrementally remove predicate symbols, as violations of assertion conditions are detected:

- we can detect when some assertion condition instance is violated (Def. 15);
- we need a way to obtain a set of assertion condition instances from predprops (anonymous asserion condition instances);

We do that by defining instantiations of anonymous assertion conditions for particular predicate symbols and the dependencies among those instances.

The following two definitions extend the notion of assertion condition instances from Def. 14 to the case of anonymous assertion conditions and higher-order literals:

Definition 21 (Labeled Hypothetical Assertion Condition). *Given a predprop $pp(X)$ and a predicate symbol $p \in \mathsf{PS}$, $\mathcal{A}^{\#}_C[pp(p)]$ denotes the set of* labeled *hypothetical assertion conditions of the form $h\#C_p$, such that $C[X] \in \mathcal{A}_C[pp(X)]$ (Def. 19), $L = X(V_1, \ldots, V_n)$, $L_p = p(V_1, \ldots, V_n)$, C_p is defined as:*

$$C_p = \begin{cases} \mathsf{calls}(L_p, Pre) & \text{if } C[X] = \mathsf{calls}(L, Pre) \\ \mathsf{success}(L_p, Pre, Post) & \text{if } C[X] = \mathsf{success}(L, Pre, Post) \end{cases}$$

and h is an identifier that is unique for each C_p.

In this semantics we allow the assertion condition instances to be derived from the hypothetical assertion conditions in the same way, as in Def. 14. However, the violation of such an instance has to be treated in a special way, as it does not signal the violation of its conditions, but instead of the corresponding predprop. For simplicity, we also introduce a special label h_0 to denote the assertion conditions that appeared originally in the program. The error set $\mathcal{E}$ in Def. 15 contained negated assertion condition instance identifiers. Now we extend this set with *assertion dependency rules* of the form $\bigwedge(\bigvee \bar{c}) \rightarrow \bar{c}$. The following definitions provide the description of how such dependencies are generated.

Definition 22 (Literal Simplification). *The* simplification *of a literal L w.r.t. θ is defined as:*

$$\mathsf{simp}(L, \theta) = \begin{cases} L & \text{if } L \text{ is a predprop} \\ true & \text{if } \theta \Rightarrow_P L \\ false & \text{if } \theta \not\Rightarrow_P L \end{cases}$$

We extend this definition for a conjunction of literals.

Definition 23 (Extension of $\mathcal{A}_C$ and $\mathcal{E}$ for dynamic predprop checking). *Given the label c of an assertion condition instance and a formula of the form $Props = \bigvee_{i=1}^{n}(\bigwedge_{j=0}^{m(i)} Prop_{ij})$, where $Prop_{ij}$ is either a prop or predprop literal, the* extension *of $\mathcal{A}_C$ and $\mathcal{E}$ for dynamic predprop checking, denoted as $\mathsf{ext}(\mathcal{A}_C, c, Props) = (\Delta\mathcal{A}_C, \Delta\mathcal{E})$, is obtained as follows:*

1. *if $\mathsf{simp}(Props, \theta) = false$, then $\Delta\mathcal{A}_C = \emptyset$ and $\Delta\mathcal{E} = \{\bar{c}\}$;*
2. *otherwise: $\Delta\mathcal{A}_C = \bigcup_{i=1}^{n} \mathcal{A}^i_C$, and $\Delta\mathcal{E} = \{\bigwedge_{i=1}^{n}(\bigvee_{h \in H_i} \bar{h}) \rightarrow \bar{c}\}$ where:*

$$\begin{aligned} \mathcal{A}^i_C = \{&h\#C \in \mathcal{A}^{\#}_C[Prop_{ij}] \mid 0 \leq j \leq m(i), Prop_{ij} = pp_{ij}(X_{ij}), \\ &pp_{ij}(X_{ij}) \text{ is a predprop and } X_{ij} \text{ is bound to some } q \in \mathsf{PS}\}. \\ H_i = \{&h \mid h\#\_ \in \mathcal{A}^i_C\}, \end{aligned}$$

We will denote as $S \rightsquigarrow_{H\mathcal{A}_\mathsf{d}} S'$ a reduction from a state S to a state S' under the current semantics.

Definition 24 (Reductions in Higher-order Programs with Higher-order Assertions). *A state $S = \langle L :: G \mid \theta \mid \mathcal{E} \rangle$, where L is a literal can be* reduced *to a state S', denoted $S \rightsquigarrow_{H\mathcal{A}_d} S'$, as follows:*

1. *If L is a constraint or $L = X(t_1, \ldots, t_n)$, then $S' = \langle G' \mid \theta' \mid \mathcal{E} \rangle$ where G' and θ' are obtained in a same manner as in $\langle L :: G \mid \theta \rangle \rightsquigarrow_{\mathcal{A}} \langle G' \mid \theta' \rangle$;*
2. *If L is an atom and $\exists (L \texttt{:-} B) \in \mathsf{defn}(L)$, then for each $c_i \# C_i \in \mathcal{A}^{\#}_C(L)$:*

$$h_i = \begin{cases} h & \text{if } C_i \text{ is an instance of some } h\#C \in \mathcal{A}_C \\ h_0 & \text{otherwise} \end{cases}$$

$$(\Delta_i\mathcal{A}_C, \Delta_i\mathcal{E}) = \begin{cases} \mathsf{ext}(\mathcal{A}_C, c_i, Pre) & \text{if } C_i = \mathsf{calls}(L, Pre) \\ (\emptyset, \emptyset) & \text{otherwise} \end{cases}$$

$$PostC_i = \begin{cases} \mathsf{check}(c_i) & \text{if } C_i = \mathsf{success}(L, Pre_i, Post_i) \\ & \quad \text{and } \mathsf{simp}(Pre_i, \theta) = true \\ true & \text{otherwise} \end{cases}$$

   and $S' = \langle B :: PostC :: G \mid \theta \mid \mathcal{E}' \rangle$, where $\mathcal{E}' = \mathcal{E} \cup \bigcup_i \{\bar{c}_i \rightarrow \bar{h}_i\} \cup \bigcup_i \Delta_i\mathcal{E}$, $\mathcal{A}'_C = \mathcal{A}_C \cup \bigcup_i \Delta_i\mathcal{A}_C$, and $PostC$ is the sequence $PostC_1 :: \ldots :: PostC_n$ (simplifying `true` *literals).*
3. *If L is a check literal $\mathsf{check}(c)$ and $c\#\mathsf{success}(L', \_, Post) \in \mathcal{A}^{\#}_C(L')$, then $S' = \langle G \mid \theta \mid \mathcal{E}' \rangle$ where $(\Delta\mathcal{E}, \Delta\mathcal{A}_C) = \mathsf{ext}(\mathcal{A}_C, c, Post)$, $\mathcal{E}' = \mathcal{E} \cup \Delta\mathcal{E}$ and $\mathcal{A}'_C = \mathcal{A}_C \cup \Delta\mathcal{A}_C$.*

Note that in this semantics we support more than one *calls* assertion condition per predicate (as several predprops may be applied to the same predicate symbol). Also note that in general we cannot prove with dynamic checking that a predprop is *true*. So, as a safe approximation we treat preconditions in such *success* assertion conditions as $false$.

Definition 25 (Trivial Assertion Condition). *An assertion condition C is* trivial *if it is of the form $\mathsf{calls}(\_, true)$ or $\mathsf{success}(\_, \_, true)$. It is also assumed that for any predprop $pp(X)$ $\mathcal{A}_C[pp(X)]$ does not contain trivial assertion conditions.*

Theorem 3 (Higher-order Run-time Checking). *For any annotated program $(P, \mathcal{Q}, \mathcal{A})$, if $\exists D \in \mathsf{derivs}_{H\mathcal{A}_d}(\mathcal{Q})$ s.t. $\neg\mathsf{rtsolve}(C, D) \Rightarrow C \in \mathcal{A}_C$ is $false$.*

Proof. In this proof we reflect the case when an assertion condition is falsified because of some of its predprops violation. To do so it is enough to show that at most one predprop was violated. Let us first prove the theorem for the case when the falsified assertion condition is $C_c = \mathsf{calls}(L, pp(X))$ and then for the case $C_s = \mathsf{success}(L, Pre, pp(X))$, where $pp(X)$ is a predprop. Without the loss of generality we assume that $\mathcal{A}_C[pp(X)]$ has cardinality of 1 (which is a case when $pp(X)$ consists of one anonymous assertion and one of the corresponding anonymous assertion conditions is trivial).

$\neg\mathsf{rtsolve}(C_c, D)$
$\Leftrightarrow$ From Def. 17: $\exists c', C'_c, \sigma, L\ (c'\#C'_c \in \mathcal{A}^{\#}_C(L)) \wedge (\sigma(C_c) = C'_c) \wedge (\mathcal{E}(D) \vdash \bar{c}')$

$\Rightarrow$ From Def. 24 and $\mathcal{E}(D) \vdash \bar{c}'$ it must hold $D = (\ldots, S_1, \ldots, S_2, S_3 \ldots, S_4, \ldots)$ where:

$S_1 = \langle L' :: \_ \mid \theta_1 \mid \_ \rangle$ s.t. $\exists\ L'\text{:-}B' \in \mathsf{defn}(L), c'\#\mathsf{calls}(L', \sigma(pp(X))) \in \mathcal{A}^{\#}_C(L)$, $\theta_1 \models (X = q), q \in \mathsf{PS}$
$S_2 = \langle L_2 :: \_ \mid \_ \mid \mathcal{E}_2 \rangle$ s.t. $\{\bar{h} \rightarrow \bar{c}', \bar{c}' \rightarrow \bar{h}_0, \} \in \mathcal{E}_2, h\#C_q \in \mathcal{A}^{\#}_C[pp(q)], L_2 = q(\ldots)$
$S_3 = \langle \_ \mid \_ \mid \mathcal{E}_3 \rangle$ s.t. $\{\bar{c}'' \rightarrow \bar{h}\} \in \mathcal{E}_3,\ c''\#C''_c \in \mathcal{A}^{\#}_C(L_2)$
$S_4 = \langle \_ \mid \_ \mid \mathcal{E}_4 \rangle$ s.t. $\mathcal{E}_4 \vdash \bar{c}''$

$\Rightarrow$ From $\mathcal{E}_3 \vdash \bar{c}''$ and Th. 2 we know that $\neg\mathsf{checked}(C''_c)$ and thus $(X = q) \notin |pp(X)|$ according to Def. 20.
$\Rightarrow$ From Def. 6 it follows that $\theta_3 \not\Rightarrow_P pp(q)$
$\Rightarrow$ Given the state S_1 before the call to L' and the state S_3: $(\mathsf{prestep}(L, D) = (\theta_3, \sigma)) \wedge (\theta' \not\Rightarrow_P \sigma(pp(X)))$
$\Rightarrow$ From Def. 9 $\neg\mathsf{solve}(C_c, D) \Rightarrow$ From Def. 11 $\mathsf{false}(C_c)$ $\square$

$\neg\mathsf{rtsolve}(C_s, D)$
$\Leftrightarrow$ From Def. 17: $\exists c', C'_s, \sigma, L\ (c'\#C'_s \in \mathcal{A}^{\#}_C(L)) \wedge (\sigma(C_s) = C'_s) \wedge (\mathcal{E}(D) \vdash \bar{c}')$
$\Rightarrow$ From Def. 24 and $\mathcal{E}(D) \vdash \bar{c}'$ it must hold $D = (\ldots, S_1, S_2, \ldots, S_3, S_4, \ldots, S_5, S_6, \ldots, S_7, \ldots)$ where:

$S_1 = \langle L' :: \_ \mid \theta_1 \mid \_ \rangle$ s.t. $\exists\ L'\text{:-}B' \in \mathsf{defn}(L)$, $c'\#\mathsf{success}(L', \sigma(Pre), \sigma(pp(X))) \in \mathcal{A}^{\#}_C(L)$, $\theta_1 \Rightarrow_P \sigma(Pre)$
$S_2 = \langle B' :: \mathsf{check}(c') :: \_ \mid \_ \mid \mathcal{E}_2 \rangle$ s.t. $\{\bar{c}' \rightarrow \bar{h}_0\} \in \mathcal{E}_2$
$S_3 = \langle \mathsf{check}(c') :: \_ \mid \_ \mid \_ \rangle$
$S_4 = \langle \_ \mid \theta_4 \mid \mathcal{E}_4 \rangle$ s.t. $\theta_4 \models (X = q), q \in \mathsf{PS}, \{\bar{h} \rightarrow \bar{c}'\} \in \mathcal{E}_4$, $h\#C_q \in \mathcal{A}^{\#}_C[pp(q)]$.
$S_5 = \langle L_5 :: \_ \mid \_ \mid \_ \rangle$ s.t. $L_5 = q(\ldots)$
$S_6 = \langle \_ \mid \_ \mid \mathcal{E}_6 \rangle$ s.t. $\{\bar{c}'' \rightarrow \bar{h}\} \in \mathcal{E}_6$ where $c''\#C''_s \in \mathcal{A}^{\#}_C(L_5)$
$S_7 = \langle \_ \mid \theta_7 \mid \mathcal{E}_7 \rangle$ s.t. $\mathcal{E}_7 \vdash \bar{c}''$

$\Rightarrow$ From $\mathcal{E}_7 \vdash \bar{c}''$ and Th. 2 we know that $\neg\mathsf{checked}(C''_s)$ and thus $(X = q) \notin |pp(X)|$ according to Def. 20.
$\Rightarrow$ From Def. 6 it follows that $\theta_7 \not\Rightarrow_P pp(q)$
$\Rightarrow$ Given the state S_1 before the call to L' and the state S_7: $(\mathsf{step}(L, D) = (\theta_1, \sigma, \theta_7)) \wedge (\theta_1 \Rightarrow_P \sigma(Pre)) \wedge (\theta_7 \not\Rightarrow_P \sigma(pp(X)))$ for $c'\#C'_s \in \mathcal{A}^{\#}_C(L)$
$\Rightarrow$ From Def. 9 $\neg\mathsf{solve}(C_s, D) \Rightarrow$ From Def. 11 $\mathsf{false}(C_s)$ $\square$

Let us trace finished derivations D^1, D^2 and D^3 from the queries $Q_1 = (\texttt{test\_c(n,X)}, true), Q_2 = (\texttt{test\_c(c,X)}, true)$ and $Q_3 = (\texttt{((test\_s(1,P),P(-2))}, true)$, respectively, to the program in Fig. 1.

In ${D^1}_{[1]}$ we encounter two assertions for `test_c/2` with a predprop in each precondition and trivial postconditions. According to state reduction rules, $\Delta\mathcal{A}_C$ consists of calls assertion condition instance c_1 and two hypothetical assertion conditions h_1 and h_2, derived from predprops `nneg/1` and `neg/1`, and $\Delta\mathcal{E} = \{\bar{c}_1 \rightarrow \bar{h}_0, \bar{h}_1 \wedge \bar{h}_2 \rightarrow \bar{c}_1\}$. In ${D^1}_{[2]}$ and current goal `P(-1)` (which is implicitly reduced

Table 1. A derivation of the query (`test_c(n,X)`, *true*) to the program in Fig. 1.

G	$\Delta\theta$	$\Delta\mathcal{E}$	Δ(labeled instances + hypothetic $\mathcal{A}_C$)
`test_c(n,X)`	$P = n$	$\bar{c}_1 \to \bar{h}_0$	c_1#calls$(test_c(n, X), nneg(n) \vee neg(n))$
	$N = -1$	$\bar{h}_1 \wedge \bar{h}_2 \to \bar{c}_1$	h_1#success$(n(Z), true, nnegint(Z))$
	$X = N$		h_2#success$(n(Z), true, negint(Z))$
`P(-1)`	$Z = -1$	$\bar{c}_2 \to \bar{h}_1$	c_2#success$(n(-1), true, nnegint(-1))$
		$\bar{c}_3 \to \bar{h}_2$	c_3#success$(n(-1), true, negint(-1))$
check(c_2), check(c_3)	-	$\bar{c}_2$	-
check(c_3)	-	-	-
$\square$	-	-	-

as `n(-1)`), success assertion condition instances c_2 and c_3 are derived from the hypotheses h_1 and h_2, and $\Delta\mathcal{E} = \{\bar{c}_2 \to \bar{h}_1, \bar{c}_3 \to \bar{h}_2\}$. Consequently, two check literals, check(2) and check(3) are added to the goal sequence. In states $D^1{}_{[3]}$ and $D^1{}_{[4]}$ those literals are reduced, which results in adding $\bar{c}_2$ to $\mathcal{E}$ because `nnegint(-1)` property from the postcondition of c_2 is violated. This example shows that the mechanism of dependencies between assertion conditions allows to avoid "false negative" results in assertion checking.

Table 2. A derivation of the query (`test_c(c,X)`, *true*) to the program in Fig. 1.

G	$\Delta\theta$	$\Delta\mathcal{E}$	Δ(labeled instances + hypothetic $\mathcal{A}_C$)
`test_c(c,X)`	$P = c$	$\bar{c}_1 \to \bar{h}_0$	c_1#calls$(test_c(c, X), nneg(c) \vee neg(c))$
	$N = a$	$\bar{h}_2 \wedge \bar{h}_3 \to \bar{c}_1$	h_2#success$(c(Z), true, nnegint(Z))$
	$X = N$		h_3#success$(c(Z), true, negint(Z))$
`P(a)`	$Z = a$	$\bar{c}_2 \to \bar{h}_2$	c_2#success$(c(a), true, nnegint(a))$
		$\bar{c}_3 \to \bar{h}_3$	c_3#success$(c(a), true, negint(a))$
check(c_2), check(c_3)	-	$\bar{c}_2$	-
check(c_3)	-	$\bar{c}_3$	-
$\square$	-	-	-

The derivation D^2 is similar to D^1. The difference is in $D^2{}_{[4]}$ state, when it becomes possible to infer $\mathcal{E} \vdash \bar{c}_1$ and thus to conclude that `c/1` $\notin |nneg(X)| \wedge$ `c/1` $\notin |neg(X)|$ and that both assertions for `test_c/2` are $false$ for this query.

In $D^3{}_{[1]}$ we encounter two assertions with a predprop in each postcondition. According to state reduction rules, $\Delta\mathcal{A}_C$ for this state consists of calls and success assertion condition instances, c_1 and c_2, $\Delta\mathcal{E} = \{\bar{c}_0 \to \bar{h}_0, \bar{c}_1 \to \bar{h}_0\}$ for them. Also, a check literal check(c_1) is added to the goal sequence. After its reduction a hypothetical assertion condition h_2, derived from `nneg(X)` predprop, is added to $\mathcal{A}_C$ in $D^3{}_{[3]}$, and $\mathcal{E}$ is extended with a dependency rule $\{\bar{h}_2 \to \bar{c}_1\}$. In state $D^3{}_{[4]}$ an assertion condition instance c_2 is obtained from h_2 and $\Delta\mathcal{E} =$

Table 3. A finished derivation of the query ((`test_s(1,P),P(-2)`), *true*) to the program in Fig. 1.

G	$\Delta\theta$	$\Delta\mathcal{E}$	Δ(labeled instances + hypothetic $\mathcal{A}_C$)
`test_s(1,P),` `P(-2)`	$N = 1$	$\bar{c}_0 \rightarrow \bar{h}_0$	c_0#calls$(test_s(1,P), nnegint(1) \vee negint(1))$
		$\bar{c}_1 \rightarrow \bar{h}_0$	c_1#success$(test_s(1,P), nnegint(1), nneg(P))$
`P = z,` `check(`c_1`),` `P(-2)`	$P = z$	-	-
`check(`c_1`),` `P(-2)`	-	$\bar{h}_2 \rightarrow \bar{c}_1$	h_2#success$(z(Z), true, nnegint(Z))$
`P(-2)`	$Z = -2$	$\bar{c}_2 \rightarrow \bar{h}_2$	c_2#success$(z(-2), true, nnegint(-2))$
check(c_2)	-	$\bar{c}_2$	-
$\square$	-	-	-

$\{\bar{c}_2 \rightarrow \bar{h}_2\}$. Finally, in state $D^3{}_{[5]}$ it becomes possible to infer $\mathcal{E} \vdash \bar{c}_1$ and thus detect that the corresponding assertion for `test_s/2` predicate is $false$ because of the predprop `nneg(X)` violation.

5 Conclusions and Future Work

This paper contributes towards filling the gap between higher-order (C)LP programs and assertion-based extensions for error detection and program validation. To this end we have defined a new class of properties, "predicate properties" (*predprops* in short), and proposed a syntax and semantics for them. These new properties can be used in assertions for higher-order predicates to describe the properties of the higher-order arguments. We have also discussed several operational semantics for performing run-time checking of programs including predprops and provided correctness results.

Our predprop properties specify conditions for predicates that are independent of the usage context. This corresponds in functional programming to the notion of *tight* contract satisfaction [26], and it contrasts with alternative approaches such as *loose* contract satisfaction [25]. In the latter, contracts are attached to higher-order arguments by implicit function wrappers. The scope of checking is local to the function evaluation. Although this is a reasonable and pragmatic solution, we believe that our approach is more general and more amenable for combination with static verification techniques. For example, avoiding wrappers allows us to remove checks (e.g., by static analysis) without altering the program semantics. [6] Moreover, our approach can easily support *loose*

[6] E.g. `f(g)=g` is not an identity function if wrappers are added to `g` on call. This complicates reasoning about the program, and may lead to unexpected and hard to detect differences in program semantics. Similar examples can be constructed where the presence of predprops in assertions would invalidate many reasonable program transformations.

contract satisfaction, since it is straightforward in our framework to optionally include wrappers as special predprops.

We have included the proposed predprop extensions in an experimental branch of the Ciao assertion language implementation. This has the immediate advantage, in addition to the enhanced checking, that it allows us to document higher-order programs in much more accurate way. We have also implemented several prototypes for operational semantics with dynamic predprop checking (see the appendix A for a minimalistic implementation), which we plan to integrate into the already existing assertion checking mechanisms for first-order assertions.

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A Minimalistic Sample Implementation

The following code (portable to most Prolog systems with minor changes) shows a minimalistic sample implementation (as an interpreter `intr/1`) of the operational semantics for dynamic predprop checking (Def. 24). Conciseness and simplicity has been favoured over efficiency. We assume that clauses, assertion conditions, and predprops have been parsed and stored in `cl/2`, `ac/1`, `pp/2` facts, respectively. The interpreter will throw an exception the first time that a failed program assertion is detected (see `ext/2` predicate). E.g., `intr((test_s(1,P),P(1)))` is a valid query while `intr((test_s(1,P),P(-2)))` throws a failed assertion exception. Predicate `reset/0` must be called between `intr/1` queries to reset error status and temporary data. In the handler errors can be gathered (as in the semantics) or execution aborted.

```
:- module(_, [reset/0, intr/1], [hiord, dcg, dynamic_clauses]).
:- use_module(library(aggregates)).

% ---------------------------------------------------------------------------
% Sample program data and properties

% negint/1 and nnegint/1 properties
eval_prop(negint(X)) :- integer(X), X < 0.
eval_prop(nnegint(X)) :- integer(X), X >= 0.

% predprops nneg/1 and neg/1
pp(nneg(P), ac(P(X), nneg_c1(P)#success(true, nnegint(X)))).
pp(neg(P), ac(P(X), neg_c1(P)#success(true, negint(X)))).

% assertion conditions and clauses for test_s/2
ac(test_s(N,_P), c1#calls((nnegint(N);negint(N)))).
ac(test_s(N,P), c2#success(nnegint(N), nneg(P))).
ac(test_s(N,P), c3#success(negint(N), neg(P))).
cl(test_s( 1,P), P = z).
cl(test_s(-1,P), P = n).

% clauses for z/1, n/1
cl(z(1), true).  cl(z(-2), true).  cl(n(-1), true).  cl(n(-2), true).

% ---------------------------------------------------------------------------
% Intepreter

:- dynamic hyp_ac/2. % hypothetical assertion condition
:- dynamic negac/1. % (negated) assertion dependency rule

% Reset errors and hypothetical assertion conditions
reset :- retractall(hyp_ac(_, _)), ( retract((negac(_) :- _)), fail ; true ).

% Interpreter with higher-order assertion checking
intr(X) :- ctog(X, X1), !, intr(X1).
```

```
intr(X) :- is_blt(X), !, X.
intr((A,B)) :- !, intr(A), intr(B).
intr((A ; B)) :- !, ( intr(A) ; intr(B) ).
intr(A) :-
    get_acs(A, Acs),
    pre(Acs, Ids, []), cl(A, Body), intr(Body), post(Ids, Acs).

% Built-ins
is_blt(true).   is_blt(fail).   is_blt(_ = _).

% From call(N,...) to N(...), where N is a predicate symbol
ctog(X, _) :- var(X), !, throw(inst_error).
ctog(X, X1) :-
    X =.. [call,N|Args],
    ( atom(N) -> true ; throw(inst_error) ),
    X1 =.. [N|Args].

% Get assertion conditions for the given literal A
get_acs(A, Acs) :- ( bagof(Ac, get_ac(A, Ac), Acs) -> true ; Acs = [] ).
get_ac(A, Ac) :- ( ac(A, Ac) ; hyp_ac(A, Ac) ).

pre([]) --> [].   pre([Ac|Acs]) --> pre_(Ac), pre(Acs).
pre_(Id#calls(Pre)) --> { ext(Pre, Id) }.
pre_(Id#success(Pre, _)) --> ( { simp0(Pre, true) } -> [Id] ; [] ).

post([], _Acs).   post([Id|Ids], Acs) :- post_(Id, Acs), post(Ids, Acs).
post_(Id, Acs) :- member(Id0#success(_Pre,Post), Acs), Id == Id0, !, ext(Post, Id).
post_(_, _).

% Check/extend assertion conditions
ext(Props, Id) :-
    simp(Props, Props2), ext_(Props2, Id),
    ( negac(A), atom(A) -> throw(failed_assertion(A)) ; true ).
ext_(true, _Id) :- !.
ext_(false, Id) :- !, assertz((negac(Id) :- true)).
ext_(Props, Id) :- acsubs(Props, Props2), assertz((negac(Id) :- Props2)).

% Add assertion dependency rules
acsubs((A,B), (A2,B2)) :- !, acsubs(A, A2), acsubs(B, B2).
acsubs((A ; B), (A2 ; B2)) :- !, acsubs(A, A2), acsubs(B, B2).
acsubs(ac(L, Id#Ac), negac(Id)) :- ctog(L, L2), assertz(hyp_ac(L2, Id#Ac)).

% Condition simplification
simp(true, R) :- !, R = true.
simp((X;Y), R) :- !, simp(X, Rx), simp(Y, Ry), or(Rx, Ry, R).
simp((X,Y), R) :- !, simp(X, Rx), simp(Y, Ry), and(Rx, Ry, R).
simp(X, R) :- pp(X, Ac), !, R = Ac.
simp(X, R) :- eval_prop(X), !, R = true.
simp(_, R) :- R = false.
```

```
% Condition simplification for success preconditions
simp0(true, R) :- !, R = true.
simp0((X,Y), R) :- !, simp0(X, Rx), simp0(Y, Ry), and(Rx, Ry, R).
simp0(X, R) :- eval_prop(X), !, R = true.
simp0(_, R) :- R = false.

or(true, _, R) :- !, R = true.     or(false, X, R) :- !, R = X.
or(_, true, R) :- !, R = true.     or(X, false, R) :- !, R = X.
or(X, Y, (X;Y)).

and(false, _, R) :- !, R = false.  and(true, X, R) :- !, R = X.
and(_, false, R) :- !, R = false.  and(X, true, R) :- !, R = X.
and(X, Y, (X,Y)).
```